MAI 2 2. a 3. cvičení - neurčitý integrál 2.
(Najděte primitivní funkce na maximálních intervalech)

1. 1. věta o substituci:

Nechť : (i) funkce f má na intervalu (a,b) primitivní funkci F (nebo- li $\int f(t)\,dt = F(t) + C$ na (a,b))

a (ii) funkce g je definovaná na intervalu (α,β), $g(\alpha,\beta) \subset (a,b)$ a g má vlastní derivaci v každém bodě z (a,b).

Pak $\int f(g(x))g'(x)\,dx = F(g(x)) + C$ na (α,β).

$\int 2xe^{x^2}\,dx$; $\int 2xe^{-x^2}\,dx$; $\int x\sin(x^2)\,dx$; $\int x^2\cos(x^3)\,dx$; $\int x\sqrt{1-x^2}\,dx$; $\int \frac{3x^2}{\sqrt{x^3+8}}\,dx$;

$\int \frac{1}{x^2}\cos\frac{1}{x}\,dx$; $\int e^x \sin(e^x)\,dx$; (*) $\int \frac{e^x}{e^{2x}+2e^x+2}\,dx$

$\int \frac{1}{x^3}\exp(\frac{1}{x^2})\,dx$; $\int \frac{1}{\sqrt{x}}\exp(\sqrt{x})\,dx$; $\int \cos x \cdot \exp(\sin x)\,dx$;

$\int \frac{1}{x\cdot\sqrt{\ln x}}\,dx$; $\int \frac{\ln^2 x}{x}\,dx$; $\int \frac{1}{x\cdot(1+\ln^2 x)}\,dx$ $\int \frac{\ln x}{x\cdot(1+\ln^4 x)}\,dx$ (stejně i $\int \frac{x}{1+x^4}\,dx$);

$\int \cos^3 x \cdot \sin x\,dx$; $\int \sin^3 x\,dx$; (*) $\int \frac{1}{\sin x}\,dx$; (*) $\int \frac{\cos^3 x}{2+\sin x}\,dx$;

Spec. $\int \frac{g'(x)}{g(x)}\,dx = \ln|g(x)| + C$ na intervalu, kde je $g(x) \neq 0$:

$\int \frac{2x}{4+x^2}\,dx$; $\int \frac{x^3}{1+x^4}\,dx$; $\int \frac{2x+4}{x^2+4x+5}\,dx$; $\int \frac{x-3}{x^2+4x+5}\,dx$; $\int \frac{1}{(1+\sqrt{x})\sqrt{x}}\,dx$;

$\int \frac{\sin x}{5+\cos x}\,dx$; $\int tgx\,dx$; $\int \frac{1}{1+tgx}\cdot\frac{1}{\cos^2 x}\,dx$; $\int \frac{\ln x}{x(1+\ln^2 x)}\,dx$.

2. 2. věta o substituci:

Funkce f je spojitá na intervalu (a,b), g' je spojitá a $g' \neq 0$ na intervalu (α,β), $g(\alpha,\beta) = (a,b)$, pak, je-li $\int f(g(t))g'(t)\,dt = G(t) + C$ na (α,β), je na intervalu (a,b) $\int f(x)\,dx = G(g^{-1}(x)) + C$:

$\int \frac{1}{x+2\sqrt{x}+2}\,dx$ ($\sqrt{x}=t$); $\int \frac{1+tg^2x}{1+tgx}\,dx$ ($tgx=t$); $\int \sqrt{1-x^2}\,dx$;

(*) $\int \sqrt{x^2+1}\,dx$ a $\int \frac{1}{\sqrt{x^2+1}}\,dx$ ($x = \sinh t\,(=\frac{e^t - e^{-t}}{2})$).

3. Integrace „per partes“ + substituce:

$$\int arctg\, x\, dx\ ;\ \int x\cdot arctg\, x\, dx\ ;\ \int \arcsin x\, dx\ ;\ \int \frac{x^2 arctg\, x}{1+x^2}\, dx\ ;\ \int \sqrt{1-x^2}\, dx\ ;$$

$$\int \ln(x+\sqrt{1+x^2})\, dx\ ;\ \int \arcsin^2 x\, dx\ .$$

$$\int \frac{1}{x^3}\exp(\frac{1}{x^2})\, dx\ ;\ \int \frac{1}{x^2} arctg(\frac{1}{x})\, dx\ ;\ \int e^{\sqrt{x}}\, dx\ ;\ \int arctg\sqrt{x}\, dx\ ;\ \int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}\, dx\ ;\ \int \arcsin^2 x\, dx\ .$$

4. Integrál z racionální funkce:

a) integrace parciálních (jednoduchých) zlomků:

$$\int \frac{1}{(x+3)^3}\, dx\ ;\ \int \frac{1}{2-x}\, dx\ ;\ \int \frac{2x+4}{x^2+4x+5}\, dx\ ;\ \int \frac{x-2}{x^2+4x+5}\, dx\ ;\ \int \frac{2x-1}{x^2+2x+5}\, dx\ ;$$

$$\int \frac{x-1}{(x^2+2x+2)^2}\, dx\ ;\ \int \frac{1}{(x^2+9)^3}\, dx\ .$$

b) integrace racionálních funkcí (rozklad na parciální zlomky a jejich integrace):

$$\int \frac{2x-11}{x^2+3x-10}\, dx\ ;\ \int \frac{3x+9}{(x^2-1)(x+2)}\, dx\ ;\ \int \frac{x^3+5x^2+15x+12}{x^2+3x+2}\, dx\ ;\ \int \frac{3x^2+2x+2}{x^3-3x-2}\, dx\ ;$$

$$\int \frac{x}{x^4-3x^2+2}\, dx\ ;\ \int \frac{x^4+1}{x^3-x^2+x-1}\, dx\ ;\ \int \frac{1}{(x^2+1)(x+1)}\, dx\ ;\ \int \frac{5x^2+2x+3}{x^3+x^2-2}\, dx$$

$$\int \frac{x^3+x^2-2x-10}{x^3+4x^2+5x}\, dx\ ;\ \int \frac{7x+4}{x^3+x^2-8x-12}\, dx\ ;\ \int \frac{x^3-x-1}{(x^2+2)^2}\, dx\ .$$